NA60 and BR Scaling In Terms of The Vector Manifestation: Formal Consideration

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The arguments developed in the preceding article on how BR scaling would predict for dilepton production in heavy-ion collisions, e.g., NA60, are augmented with more precise and rigorous arguments.

In the preceding article [1], we gave an extremely simplified model argument why the green curve erroneously attributed to BR scaling in the NA60 results [2] has nothing to do with what we consider to be BR scaling proposed in 1991 [3] and modernized recently with hidden local symmetry theory of Harada and Yamawaki [4] with vector manifestation [5]. In order to make our arguments accessible to non-theorists, we used in [1] arguments that are somewhat over-simplified and hence lacking rigor and precision. In this paper, we supply the missing details with more precise definitions and reasoning, indicating which arguments are rigorous and which are not and what needs to be done to make them firmer. The conclusions presented in [1] remain qualitatively unchanged although some were heuristic and incomplete, so the readers who are convinced by the arguments given there could skip reading this paper.

In our approach to the physics of matter under extreme conditions, the correct implementation of the basic premise of BR scaling is mandatory not only to the immediate problem of theoretically interpreting the NA60 data or other dilepton data but also to the whole gamut of RHIC physics since the vector manifestation (VM in short, see below) of chiral symmetry discovered in hidden local symmetry theory by Harada and Yamawaki [5], we believe, is relevant not only in the hadronic phase into which the high temperature and high density matter produced in relativistic heavy-ion collisions or superdense matter formed in gravitational collapse in compact statrs evolves but also for understanding the structure of the "new form" of matter found in the chirally restored phase as discussed in a series of recent papers by Brown et al. addressing RHIC physics [6, 7, 8]. In this paper we will focus on aspects related to dilepton production.

There are three key elements of hidden local symmetry theory with vector manifestation (HLS^{VM} in short, to be distinguished from a more general HLS theory described below) relevant to dilepton production in heavy-ion collisions: (1) vector dominance (VD) in the photon coupling to matter, (2) the distinction and role of the "parametric" mass and the physical (or pole) mass of the vector meson and (3) the "sobar" excitations and "fusing" of BR scaling and sobar configurations. All three are essential in describing dilepton production but have not been properly taken into account in most of the works available in the literature.

- Vector dominance (VD).

The principal point of vector dominance in the process at issue is that vector dominance is mostly violated when the photon couples to matter in medium. For this we need to understand when VD is operative and when it is not. Let us explain how this issue comes about.

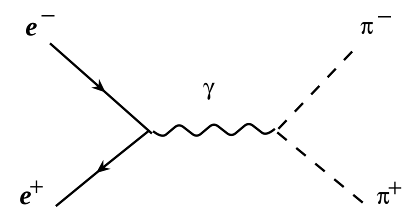

FIG. 1: Direct photon coupling to the pions which is absent in the holographic dual QCD theory and also in Harada-Yamawaki's HLS theory for $a = 2$.

In a recent important development in holographic dual QCD [9] (an approach to QCD emerging from AdS/CFT duality in string theory), low-energy QCD is given by a multiplet of (pseudo)Goldtone bosons and an infinite tower of massive vector bosons with local gauge invariant coupling. The infinite tower results from the 5-D Yang-Mills theory given by the dual bulk sector when the 5-D nonabelian gauge theory is compactified à la Kaluza-Klein to 4-D gauge invariant theory. This is a generalized hidden local symmetry theory in the sense that an infinite tower of gauge fields are involved. Let us call this theory HLS^{AdS}. It turns out that the 4-D theory is entirely vector-dominated. What is significant for our case is that the photon coupling to hadrons is *completely* vector-dominated. That means that the direct photon coupling to charged pions as depicted in Fig.1 is simply absent in the theory. However when truncated at a finite number of vector mesons, say, ρ, ω, ϕ, it is natural to expect that vector dominance is lost in general. However surprisingly while it is lost in certain processes (e.g., the EM form factor of the nucleon to be discussed below), it is preserved in some others (e.g., the $\omega \to 3\pi$ decay and some anomalous photon processes mediated by the Wess-Zumino term). In Harada-Yamawaki theory HLS^{VM}, one makes the truncation at the lowest members of the tower, ρ, ω and ϕ and integrates out all the rest of the infinite tower at some scale Λ_M and makes the Wilsonian match-

ing to QCD at Λ_M for an ultraviolet completion. In such a theory the vector dominance is of course necessarily incomplete or violated. Now the theory has three key constants (apart from quark masses etc), the gauge coupling g, F_π which is connected to the pion decay constant f_π and a which is the ratio $(F_\sigma/F_\pi)^2$ – where F_σ is the decay constant of the would-be Goldstone boson that makes up the longitudinal component of the massive vector meson. The renormalization group equation (RGE) trajectories for the constants in this HLS^{VM} theory are in general quite complex. However when constrained to QCD by matching of the correlators, things become very simple: the RGE for the gauge coupling g flows to the fixed point $g = 0$ and that for the constant a flows to 1 whereas unconstrained, they can flow to a variety of fixed points. It is the constraint imposed by matching with QCD that leads uniquely to the fixed points $(g^*, a^*) = (0, 1)$. As for F_π, the constraint with QCD does not lead to any special value in temperature (while it goes to zero in density). However the physical pion decay constant f_π related to F_π with quadratically divergent loop corrections does go to zero at the fixed point $(g^*, a^*) = (0, 1)$. This fixed point to which hadronic system is driven when chiral restoration is reached is called "vector manifestation fixed point."

We do not know whether – and how – one can probe what corresponds to the VM fixed point in HLS^{AdS}. That requires higher order computations in the bulk sector which nobody knows how to do. We shall therefore consider Harada-Yamawaki HLS^{VM} theory. For this, let us model what happens using only the low-mass mesons π, ρ etc in phenomenology which should be known to everyone. In HLS^{VM}, the photon coupling to hadrons is given by

$$\delta\mathcal{L} = -2eagF_\pi^2 A^\mu \mathrm{Tr}[\rho_\mu Q] + 2ie(1 - a/2)A^\mu \mathrm{Tr}[J_\mu Q] \quad (1)$$

where A_μ is the photon field, ρ_μ stands for the vector fields ρ, ω and ϕ, J_μ is the vector current made up of the chiral field, i.e., pions and Q is the charge matrix $Q = \frac{1}{3}\mathrm{diag}(2 - 1 - 1)$. Now the vector dominance is achieved in this theory when $a = 2$ for which the second term in (1) which corresponds to Fig.1 vanishes. However as shown by Harada and Yamawaki [4], $a = 2$ is *not* on a stable trajectory of the RGE for a and that nature is realized by vector dominance in pionic processes is merely an "accident" rather than required by QCD. In fact generic hadronic systems tend to quickly go to $a = 1$ under normal conditions as discussed in [10] which implies in particular a 50/50 electromagnetic coupling [11, 12] of hadrons directly and through the vector meson to leptons. This phenomenon is known to take place in even free space. For instance, the nucleon form factor is dipole in nature, not monopole as would be given by vector dominance. This and other phenomena discussed in [10, 11] show the vector dominance used at zero density and temperature to be very fragile and easily violated in the presence of matter and temperature. Indeed Harada and Sasaki have shown that in a heat bath, vector dominance is maximally violated with the constant a going to 1 [13]. This is not surprising since vector dominance does not appear on the RG required by QCD.

In heavy-ion processes, temperature and density are intricately correlated, so it is difficult to make a precise analysis of how a evolves as a function of *both* temperature and density. But we expect that the combination of the two will speed up the flow of a to 1 compared with the influence of either alone. What this means specifically in dilepton processes is that the vector dominance part of the Lagrangian has coefficient 1/2 as compared with the one used by most workers so that the dilepton production should be 1/4 of that calculated using vector dominance. The part of the Lagrangian in which the hadrons couple directly to dileptons will provide a background of the ρ-mesons, because the latter is not involved in the interaction.

The conclusion that we can draw from the above argument is that the overall use of vector dominance which has been the common practice in the field would lead to a curve in dilepton production as much as 4 times higher than would be given by the Lagrangian (1) that is predicted by Harada-Yamawaki HLS^{VM} theory.

• Intrinsic background dependence (IBD) and parametric mass.

In HLS^{VM} theory, the pole mass of the vector meson that appears in the current-current correlation functions measured by the experiment is given by two terms: one the "bare" or "parametric" mass and the other the thermal (and/or dense) loop-correction term, all of which are temperature/density dependent,

$$m_V = m_{bare} + \Delta M \quad (2)$$

with

$$m_{bare} = \sqrt{a} g F_\pi. \quad (3)$$

Now the first term m_{bare} consists of the parameters that figure in the bare Lagrangian which is determined in HLS^{VM} at the matching scale Λ_M whereas the second term is a loop correction computed with the same parameters. Since the correlators are Wilsonian-matched between the effective field theory (EFT) sector and QCD sector, the parameters in the EFT sector can be expressed in terms of the quantities that appear in the QCD sector, namely color-gauge coupling, quark and gluon condensates. If the matching is done in medium, then the condensates will inevitably depend on the background that defines the medium, namely, density, temperature etc. This means that the parameters a, g and F_π in the EFT sector will depend on the same background. This is referred to as "intrinsic background dependence (IBD)." When we say "BR scaling mass," we mean m_{bare}, not m_V (except near the critical point). It is this IBD term that is directly locked to the chiral symmetry property of the vacuum. Near the critical point, HLS^{VM} says that both m_{bare} and m_V go proportional to the quark condensate $\langle\bar{q}q\rangle$ and hence go to zero as BR scaling does.

If one were to do a naive tree-order calculation and ignore loop corrections, only the first term would contribute. If this were the entire story (or dominant story), the physical vector meson mass would then be following the quark condensate. However this is certainly wrong in hot matter as pointed out very clearly by Harada and Sasaki [13] and also near chiral restoration in dense matter as pointed out by Harada et al [14]. Let us focus here on the temperature effect. We will return to the density case later. In a heat bath even at low temperature, the (second) loop corrections are mandatory for consistency with the symmetry of QCD. In fact it is in combination of the two terms that the pole mass of the vector meson *increases* $\propto T^4$ near zero temperature with no T^2 term present as required by low-energy theorem [15]. Thus on the one hand, the IBD is required by the matching of the correlators at T_C to QCD [13] and on the other hand, the IBD is *not* what is measured directly in experiments. As T_c is approached, both the bare mass term and the loop corrections go to zero $\propto \langle\bar{q}q\rangle \to 0$. In this case the pole mass does directly reflect on chiral structure as does BR scaling. *Only in the vicinity of T_c does BR scaling manifest itself transparently in the pole mass of the vector meson in a heat bath.*

Now what do we know about the temperature dependence of m_{bare} and m_V? While it is clear that the vector meson pole mass will not simply follow the order parameter of chiral symmetry as the m_{bare} does in HLS^{VM}, we have no theoretical information as to what happens away from $T = 0$ and $T = T_c$. Nobody has calculated it yet although it is a doable calculation. Fortunately lattice calculations can provide the necessary information. As described in [7], we learn from Miller's lattice calculation of the gluon condensate [16] that the soft glue starts to melt at $T \approx 125$ MeV. The melting of the soft glue, which breaks scale invariance as well as chiral invariance *dynamically* – and is responsible for the hadron mass – is completed by T_c at which the particles have gone massless. The gluon condensate which remains unmelted above $T \sim T_c$ represents the hard glue, or what is called "epoxy," which breaks scale invariance *explicitly* but has no effect on the (dynamically generated) hadron mass. Coming down in temperature in heavy ion processes, $T \approx 125$ MeV is therefore the "flash" temperature at which the vector meson recovers 95% of its free-space mass. We see that the melting of the soft glue is roughly linear, implying that the meson masses increase linearly from near zero at T_c to the flash temperature. This then suggests that going up in temperature, nothing much happens to m_V until $T \sim 125$ MeV. It is this scenario that provides an extremely simple explanation [8] for the observed STAR ρ^0/π^- ratio and HBT [17].

In fact, an approximate calculation [18] of the thermal loop terms based on the work of Harada and Shibata [19] finds that up to the flash temperature of $T \sim 125$ MeV, the ΔM term in (2) is positive and small in magnitude, $\sim +10$ MeV. Since the bare mass m_{bare} is flat up to that temperature as indicated from the lattice results, it is very reasonable to assume that the vector meson pole mass does not change appreciably up to the flash point.

Let us consider now the density effect. The density probed in the dilepton experiments is in the vicinity of nuclear matter density so it is not high. Now in the bare mass term, we expect that $\sqrt{a}g$ changes little up to nuclear matter density [12], so

$$\Phi \equiv \frac{m^*_{bare}}{m_{bare}} \approx \frac{f^*_\pi}{f_\pi} \tag{4}$$

At $n \approx n_0$, we know that this should be ~ 0.8 [20] and expect that slightly above n_0 which is reached by the dilepton processes, the linear drop $\Phi(n) \approx 1 - 0.2(n/n_0)$ would be reasonable. The dense loop term ΔM is also positive and small as in the case of temperature in the regime of the density involved and it will go to zero at the VM fixed point. This means that the mass drop in density is expected to be *less* than 20% at normal nuclear matter density. This is indeed what one finds at $T \sim 0$ in the CBELSA/TAPS experiment [21] on the ω meson as well as the KEK experiment [22] on ρ and ω.

The conclusion is that combining the temperature effect and density effect, one expects the mass shift in the vector mesons in the NA60 experiment to be small, if any. *This does not mean that BR scaling is absent in the process.*

• Fusion with "sobar" configurations.

In the HLS^{AdS} theory given by holographic dual QCD, low-energy physics is entirely described by a multiplet of pseudo-Goldstone pseudoscalars, an infinite tower of massless vector bosons and a multiplet of Goldtone scalar bosons that are to be higgsed. In unitary gauge, the vector mesons become massive with the scalars eaten up. Since this represents the entire story of QCD at low energy with no fermions in the picture, we must conclude that baryons *must* emerge as skyrmions. This has been pointed out by several theorists [9, 23, 24]. It is clear what one should do: one should calculate spectral functions measured in experiments either from this generalized hidden local symmetry theory with an infinite tower of vector mesons or more practically from the Harada-Yamawaki HLS^{VM} of the lowest vectors matched to QCD. The first serious effort was made in [25] along this line using the Skyrme model that contains only the pion field and an initial attempt is being made with HLS^{VM} theory but we are still very far from a realistic calculation that can be confronted with experiments. As mentioned, if the temperature is near T_c where the vector meson mass becomes comparable to the pion mass, then Harada-Yamawaki HLS/VM theory should be reliable enough. See [10, 26] for more discussions on this point. However near $T \sim 0$ or far away from T_c, Harada-Yamawaki's HLS theory without fermion (baryon) degrees of freedom cannot fully capture the physics of all channels. For instance, it cannot describe meson condensations in dense matter unless baryon degrees of freedom are incorporated. This is because we cannot ignore certain low-energy particle-hole excitations that have the

same quantum numbers as the mesonic degrees of freedom we are looking at. Notable examples are the "sobar" excitations, e.g., the Δ-hole excitations at about 300 MeV in the pion channel, the $N^*(1520)$-hole at about 580 MeV in the ρ channel etc. As in condensed matter physics, these excitations are expected to be very important at temperature or density at which HLS/VM is not of dominant influence. Incorporating these sobar configurations in studying spectral functions is referred to as "fusing" [12, 27]. How to treat the fusing of sobar degrees of freedom with elementary excitations was first discussed in [28] using a field theory technique with a toy model. If one were to generate baryons N, N^* etc from HLS theory as skyrmions and do an effective field theory à la Harada and Yamawaki, then the intrinsic density/temperature dependence that underlies BR scaling would be well-defined and hence the fusing could be done in a consistent way. This has not been done yet. Positing the presence of the baryons and the Fermi surface as was done in [12, 27] is undoubtedly *ad hoc* but that's the best one can do at the moment.

The above caveat notwithstanding, our assertion is that in describing dilepton process such NA60, BR scaling reflecting on chiral symmetry in hot matter that is probed – which is a parametric property of the effective Lagrangian – should be implemented in the fusing of the sobar and elementary configurations. This phenomenon cannot be captured *except perhaps very near the critical point* by just a single elementary field with a dropping mass as has been done so far. This aspect has been amply emphasized in review articles [12, 27].

In short, it would be too naive to expect that the shape of the spectral function measured in the NA60 provide a *direct* information on the chiral structure of the hadronic system. A mere shift of the peak either way cannot be taken as a signal for or against BR scaling.

- "Seeing" chiral restoration, partial or full.

There is an intense effort among both theorists and experimenters to find a direct signal for both what is called "partial" chiral restoration and "full" chiral restoration. The former is looked for in what are considered to be "physical" in-medium quantities like pion decay constant, vector meson mass etc. and the latter in probing matter under extreme conditions such as in heavy ion collisions or in compact stars.

Needless to say, what one measures in experiments are correlation functions, not such theoretical quantities like in-medium mass, in-medium decay constant etc. How to interpret the behavior of masses and coupling constants in one's theory depend on what theory one is using. Now in connection with chiral symmetry one would like to have a set of parameters that reflect in a *known way* on the order parameter of the symmetry such as for instance the quark condensate in the case of chiral symmetry in QCD and to express in a consistent way the correlators one would like to study. To learn how chiral symmetry manifests itself as the conditions in the vacuum are changed, one would have to map out the parameters that are locked in a known way to the order parameter, i.e., quark condensate. BR scaling is one such parametrization, not necessarily the only one. They are the ones which will ultimately provide the desired information. It is not necessarily physical variables themselves that will do so. The theoretical situation is much clearer when one is near the phase transition, as we stated, thanks to the vector manifestation in the case of HLS^{VM} theory. But it cannot be so away from the transition point.

To illustrate that one can easily arrive at a completely wrong picture if one ignores subtlety involved in the notion of hadron effective mass – whether it is BR scaling mass or Landau effective mass, let us look at the well-known case of the EM orbital current and the in-medium mass of a nucleon sitting on top of the Fermi sea which was in the past cited as an evidence against BR scaling. Consider a chiral Lagrangian in which BR scaling is suitably implemented. A nucleon in nuclear matter described as a quasi-particle with such a Lagrangian will carry a BR scaling mass m^* which drops as density increases in some proportion to the quark condensate. Assuming as is often done in nuclear physics that the impulse approximation is valid in response to the slowly varying EM field, one may write the isoscalar convection current for the nucleon with the momentum $\mathbf{k}$ as [29]

$$\mathbf{J} = e\frac{\mathbf{k}}{2m^*}. \tag{5}$$

Now suppose that experimentalists measure the isoscalar orbital gyromagnetic ratio g_l^0 defined by

$$\mathbf{J} = eg_l^0\mathbf{k}/m_N \tag{6}$$

where m_N is the free-space proton mass $m_N = 938$ MeV. Their experiment will of course yield (as we all know)

$$g_l^0 = 1/2 \tag{7}$$

and *not* $g_l^0 = \frac{1}{2}(m_N/m*) > 1/2$, as the naive calculation would give. They might then be tempted to say "BR scaling is ruled out by the experiment on the gyromagnetic ratio in nuclei."

This conclusion is completely wrong. In Fermi liquid theory, one gets the convection current given by the free-space mass due to a well-known mechanism, variously attributed to charge conservation, Galilei invariance etc. In condensed matter physics it is related to Kohn theorem for cyclotron frequency of an electron. In many-body theory language, it is the effect of "back-flow." In the case with BR scaling in a chiral Lagrangian, it is the chiral Ward identity that assures the correct answer as shown in [20].

This example illustrates the danger in jumping to a conclusion prematurely. There are other such cases. For example, a similar misleading conclusion could have been arrived at in spin observables in proton-nuclear scattering as discussed in [12].

- Concluding remarks.

We have discussed in a more precise language than in the preceding article three important features that characterize BR scaling as interpreted in terms of hidden local symmetry theory with vector manifestation, with a focus on their role in describing the NA60 dilepton process. First, the parameter a going to 1 in hot and dense matter will maximally violate the vector dominance and hence cut down the dilepton cross section. Secondly parameterizing the temperature dependence of the pole mass of the vector meson m_V^*/m_V as $(1-(T/T_c)^n)^d$ where d is some positive number and n an integer so that the vector mass goes to zero at $T=T_c$ as has been done by workers in the field overestimates the drop caused by temperature since the vector mass must stay more less unchanged until the temperature reaches the flash temperature ~ 125 MeV. Thirdly the fusing will have a compensating effect, e.g., the quantum mechanical level repulsion, between the "sobar" configuration and the "elementary" mode, both subject to the constraints by vector manifestation, which could shift the peak away from the naive tree-level pole position.

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